UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

AD-A248 596

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution unlimited
	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 92 0214
6a. NAME OF PERFORMING ORGANIZATION University of Exeter	6b. OFFICE SYMBOL (If applicable)
7a. NAME OF MONITORING ORGANIZATION European Office of Aerospace Research and Development	
6c. ADDRESS (City, State, and ZIP Code) Mathematics Department North Park Road Exeter EX4 4QE	7b. ADDRESS (City, State, and ZIP Code) Box 14 FPO New York 09510
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR	8b. OFFICE SYMBOL (If applicable) NM
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR 89-0042	
8c. ADDRESS (City, State, and ZIP Code) BLDG 410 Bolling AFB, DC 20332	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A4	

11. TITLE (Include Security Classification)

INSTABILITY AND TRANSITION IN COMPRESSIBLE BOUNDARY LAYERS (u)

12. PERSONAL AUTHOR(S) Professor Philip Hall

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
FINAL	FROM 11-1-88 to 10/31/91	31/12/91	13

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Hypersonic Boundary Layer, Gortler Vortex Instability

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Results are presented for (a) Real gas effects abd Gortler vortices, (b) Nonparallel effects on Gortler vortex growth of Sutherland Law fluids, (c) Secondary instability analysis of finite amplitude Gortler vortices at typersonic speeds, (d) Effects of crossflow velocities on Gortler vortices, (e) Mode interactions in compressible flows, (f) Receptivity theory for Gortler vortices.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Arje Nachman	22b. TELEPHONE (Include Area Code) (202)767-5028 — 22c. OFFICE SYMBOL AFOSR/NM

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

FINAL SCIENTIFIC REPORT

INSTABILITY AND TRANSITION IN COMPRESSIBLE FLOWS.

Philip Hall

TABLE OF CONTENTS

1. Introduction

Most of the funds associated with the contract have been used to support Dr. Y. Fu a postdoctoral research fellow at Exeter University. A number of seminars based on the research in the report have been given at several institutions whilst aspects of the research have been presented at the EUROMECH conference on Görtler vortices (Nantes) and the IUTAM symposium on nonlinear instability theory (Nice), a full list is given as an appendix. In the next twelve months invited talks on the work will be given at international conferences in Spain (IUTAM) and Columbus. Ohio (NATO). A bonus to the research program was the contribution of Dr. Nic Blackaby (a postdoctoral fellow supported jointly by ICASE and Manchester Uni.). Dr Blackaby is an expert in aspects of Rayleigh and Tollmien-Schlichting instabilities in hypersonic flows and we expect further interaction with him in the next twelve months. Further collaboration from Dr. Stephen Cowley (Imperial College), Dr Mujeeb Malik, Dr. P. Balakumar (NASA Langley) also greatly enhanced the research effort. A further bonus to the program was the award of a SERC Senior Fellowship to Professor Philip Hall, this enabled Professor Hall to concentrate

on research for a period of five years from 1.1.90. A major part of the research plan submitted for this five year period concerns instability and transition in compressible flows. In the 3 year grant period twelve months significant progress has been made towards our understanding of several important features of the transition process in compressible boundary layers. Most of the results we have found concern hypersonic boundary layers, but important results have also been obtained for subsonic flows of practical importance. In fact the results obtained for subsonic flows concern the effects of :

a **The destruction of the Görtler mechanism by a crossflow velocity field.**

b **The secondary instability of large amplitude vortices to Rayleigh instability waves.**

c **The receptivity problem for Görtler vortices**

The above investigations are clearly relevant to the hypersonic regime and progress has been made with their extensions into that situation: we shall return to discuss that work later. However our principal results which we report on here refer to linear and nonlinear aspects of hypersonic boundary layer instability theory when curvature effects are important so that instability is dominated by the Görtler instability mechanism. More precisely we have made significant progress towards the following major aims highlighted in the original research program:

A The effect of the viscosity law on the Görtler mechanism.

In particular we have shown here that the nature of the instability depends crucially on whether the Chapman or Sutherland viscosity law is used . We have shown that when the more realistic Sutherland Law is used nonparallel effects play a significant effect on the onset of the Görtler mechanism.

B Real Gas Effects and the Görtler mechanism

For a realistic viscosity law for a hypersonic boundary layer we have allowed for the effect of gas dissociation on the onset of the Görtler vortex instability mechanism. We find that Real Gas effects play a minor role in the mechanism at low wavenumbers for which the instability is concentrated in the main part of the boundary layer where the downstream velocity component approaches algebraically its free-stream value. In fact in this regime Real Gas effects could sensibly be totally ignored since they alter the growth rates of the vortices by at most a few percent. However at wavelengths comparable with the temperature adjustment layer at the edge of the boundary layer Real Gas effects have a more

significant effect. Typically we find that the growth rates of the vortices are increased by about ten percent. It could be argued that this is also perhaps an effect which might sensibly be ignored since the effect on the growth rate is not great, we will return to this point later when we discuss our results in more detail. Note here that in an ??? report we stated that Real Gas effects are stabilizing, that conclusion was based on calculations in a region of parameter space not relevant to high altitudes, at such altitudes there is certainly a destabilizing effect.

C Fully nonlinear vortex instabilies and their secondary instability to wavy vortex disturbances.

This work has in the first instance been carried out neglecting Real Gas effects and concerns the finite amplitude vortex states which exist when the basic hypersonic boundary layer is massively unstable. Remarkably we find that even then the vortex instability can be calculated asymptotically even though the original boundary layer is completely restructured by the disturbance. We show that the finite amplitude disturbance is confined to the temperature adjustment layer and that at sufficiently large amplitudes the flow suffers a secondary instability to a wavy vortex mode. This calculation is particularly important because it describes a particular route to transition in a hypersonic boundary layer. An alternative route to transition based on a secondary instability associated with a Rayleigh instability is also possible and is the subject of work in progress.

D Inviscid instability of the hypersonic boundary layer over a flat plate

In this calculation we show how the Rayleigh instability mechanism occurs in the hypersonic boundary layer on a flat plate, in this flow there is of course an attached shock and we determine the effect of the shock on the flow stability. This calculation is important because in many practical situations where the Görtler mechanism exists there are shocks present and the possible interaction of vortices and Rayleigh instability waves might cause the premature onset of transition.

In the following section we shall discuss the results obtained concerning the Görtler instability mechanism at hypersonic speeds. In the Section 3 we shall describe the results we have obtained for boundary layer flows at subsonic speeds, in the final section we shall discuss how that work can be altered to allow for hypersonic effects.

Effects of crossflows on hypersonic boundary layers

We have determined the effect of a spanwise velocity component on a hypersonic boundary later. In particular we show that inviscid hypersonic Gortler vortices are greatly stabilized by a crossflow.

2 LINEAR AND NONLINEAR INSTABILITIES IN HYPERSONIC BOUNDARY LAYERS

A The effect of the viscosity law on the Görtler mechanism.

Here we investigate the effect of a realistic (Sutherland) viscosity law on the onset of Görtler vortices. We find that the basic hypersonic boundary layer in this case splits up into a wall layer of depth $M^{\frac{3}{2}}$ in physical variables where M is the Mach number and a temperature adjustment layer of depth $O(1)$ sitting on top of the wall layer. In the wall layer the downstream velocity component varies from zero at the wall to almost its free-stream value. In the wall layer the temperature is very large, $O(M^2)$, but at the edge of the wall layer it decays algebraically and in the adjustment layer the temperature becomes $O(1)$. We find that the normal velocity component at the edge of the boundary layer is $O(M^{\frac{3}{2}})$ so that in the absence of wall curvature the basic state is massively stable to Görtler vortices because the streamline curavture of the basic state is not in the unstable sense according to Rayleigh's criterion. In order to counter this large stabilizing effect the wall must have large concave curvature and the associated Görtler number must be therefore of size $O(M^{\frac{3}{2}})$ and at zeroth order exactly counterbalance the curvature effects of the basic state. The amount by which the Görtler number must be increased above this value in order to cause instability depends on the wavelength of the imposed vortex field. If the wavelength is comparable to the wall-layer the disturbance evolves in a completely non-parallel but viscous manner and can only be described by a numerical solution of the governing pde's. The calculations we carried out in this regime show that the neutral Görtler number in this case is $M^{\frac{3}{2}}\tilde{G}$ where $\tilde{G}$ is a function of the initial disturbance and the vortex wavenumber. At sufficiently small wavenumbers $\tilde{G}$ tends to infinity whilst at sufficiently large wavenumbers it tends to the constant which enables wall curvature to exactly counterbalance the effect of curvature of the basic state. Thus in the wall layer regime the neutral Görtler number decreases monotonically from infinity to a constant value.

When the vortex wavenumber becomes large within the wall layer formulation the vortex moves towards the edge of the boundary layer and in fact when the vortex wavelength decreases to a size comparable with the adjustment layer thickness a new regime emerges. In this new regime the vortex is trapped in the adjustment layer and viscous effects are negligible. The zeroth order neutral Görtler

number again exactly counterbalances the curvature of the basic state and the next correction to it is $O(M)$ and is determined by nonparallel effects. At even higher wavenumbers viscous effects come back into play and when the wavenumber is $O(M^{\frac{1}{4}})$ the neutral Görtler number begins to increase monotonically with trhe wavenumber. Finally when the wavenumber is $OM^{\frac{3}{8}})$ viscous effects dominate and we obtain a right hand branch to the neutral curve determined correctly by a quasi-parallel theory which has the Görtler number proportional to the wavenumber to the fourth power.

In Fu, Hall and Blackaby (1991) we also made a detailed investigation of the effect of dissociation on the instability mechanism. In fact we considered a pure diatomic Lighthill gas and we showed that the key role of dissociation is to modify the curvature of the streamlines of the basic hypersonic boundary layer. For the wall mode this has an essentially negligible effect on the growth rate of a disturbance but for the adjustment layer mode changes in growth rate of the order of 10 percent are possible. In fact Real Gas effects are shown to be destabilizing in this regime whereas for the wall mode they can be either stabilizing or destabilizing. In addition the effect of wall cooling on the Görtler mechanism at hypersonic speeds was investigated. This is a topic of practical importance because at high Mach numbers enormous temperatures are generated so wall cooling would be essential in for example the NASP configuration. We find that wall cooling can have a very large effect on the disturbance growth rate in the most unstable adjustment layer mode regime; more precisely we find that the growth rate of a vortex can be increased by up to 50 percent from its adiabatic value. The significance of this is that if wall cooling is to be operational in a very high Mach number regime then designers should take account of the increased likelihood of Görtler vortex induced transition. Another possibly very important result of our investigation of relevance in design work is the observation that vortices develop in a unique manner when the wall curvature varies like the $\frac{3}{2}$ power of downstream variable. This situation is unique because the basic flow's intrinsic stabilizing curvature and the wall curvature now balance over an $O(1)$ lenghthscale in the downstream direction, smaller growth rates are achieved. This suggests to us that in practical situations where wall curvature is unavoidable, eg engine inlets, near control surfaces, the curvature can be optimally distributed to avoid transition due to Görtler vortices. This work was presented at the NICE IUTAM and will be published in the proceedings of that meeting. The work was also presented at the Görtler vortex Euromech which took place in Nantes in June 1990.

B Real Gas Effects and the Görtler mechanism

The major results we found were given in the above section; essentially we find that in the linear regime the wall mode can be either stabilized or destabilized by Real Gas effects though the associated changes in the growth rates are small. However the adjustment layer mode is destabilized more significantly by such effects though increases in growth rates by about 10 percent are the most which we found. At present we are investigating whether Real Gas effects are more significant in the nonlinear regime, this work will be discussed in the next report.

C Fully nonlinear vortex instabilies and their secondary instability to wavy vortex disturbances.

The work we report on here is based on the incompressible analysis of Hall and Lakin (PRS(A)),1988 though for a compressible boundary layer significant differences arise because the temperature field as well as the basic velocity field is restructured by the disturbance. We recall here that the Hall-Lakin description of nonlinear vortex instabilities is capable of describing analytically disturbances so large that they completely alter at zeroth order the boundary layer in question. In fact in the Hall-Lakin theory the basic state is determined as a solvability condition on a set of disturbance equations, essentially the basic boundary layer adjust itself so as to make all modes neutral; the structure found by Hall and Lakin has remarkable similarities with the so-called marginal theory of turbulence postulated some years ago by Malkus.

In Fu and Hall (1991) we use the ideas of Hall and Lakin to describe the fully nonlinear structure of small wavelength adjustment layer vortices. If the wavelength of the vortex is comparable to the thickness of the adjustmnet layer no analytical progress is possible and a numerical approach must be used. For the small wavelength vortices we show that the vortices drive the mean state in an $O(1)$ fraction of the main adjustment layer. The region of vortex activity is trapped by thin shear layers where viscous effects come more significantly into play and cause the decay of the vortices. We show that these layers are particularly important as sites of secondary instability and that they are thus unstable to wavy vortex modes which are essentially unsteady three-dimensional disturbances. Our results suggest that in a hypersonic boundary layer the initial onset of vortex activity will be in the adjustment layer, as the flow becomes more unstable nonlinear effects come into play and eventually the Hall-Lakin regime is set up. At this stage the flow is steady but the onset of the secondary wavy vortex mode leads to the onset

of a three-dimensional unsteady flow which we believe signals the onset of transition.

D Inviscid instability of the hypersonic boundary layer over a flat plate

This work can be found in Blackaby, Cowley and Hall (1991). This calculation concerns the Rayleigh instability of a flat plate boundary layer of a Sutherland law fluid. In most practical situations both Görtler vortices and Rayleigh waves will be excited so it is important for us to understand the structure of this mode in a hypersonic boundary layer. The basic state under investigation here is more complex because of the presence of a shock attached to the leading edge of the plate. Sufficiently far downstream the effect of the shock is not felt and we have a Blasius flow. Near the leading edge the shock has a massive effect on the basic boundary layer which is therefore quite unlike a Blasius flow. Between these two regimes we have a strong interaction region where the basic state can only be found numerically. We show that the leading edge and far downstream regimes are susceptible to two types of Rayleigh wave. The first type we refer to as an acoustic wave and it correspond to a disturbance of wavelength comparable to the depth of the adjustment layer where it is indeed concentrated. The second so-called vorticity mode is trapped in the adjustment layer and is usually the most unstable possible type of Rayleigh wave. This is a crucial result because the Görtler mechanism is at it's most dangerous when it is confined to the adjustment layer so we can expect a strong interaction between vortices and Rayleigh waves in that layer. We have made some progress with that problem and will return to it in the final section.

E Crossflow effects on the growth rate of inviscid Görtler vortices in a hypersonic boundary layer

The crossflow effects on the inviscid growth rate of Görtler vortices in a hypersonic boundary layer with pressure gradient is studied in Fu and Hall (1992). Attention is put on the inviscid mode trapped in the temperature adjustment layer which has greater growth rate than any other mode. It is shown that a crossflow of order $0(R^{-1/2})$ times the streamwise flow has a negligible effect on the growth rate of the inviscid Görtler vortices. However this is not the case when crossflow becomes comparable with the streamwise flow. In the latter case, it is shown that the growth rate is determined by an eigenvalue problem, which is solved numerically and analytically in several asymptotic limits. The importance of

a singular point is clarified. In the moderate wavenumber regime, it is found that as the crossflow is increased gradually from zero, the first mode eigenfunctions will hit this singular point when the crossflow reachs a critical value. For crossflows above this critical value, the eigenfunctions are always terminate by the singular point. As teh crossflow is further increased, Görtler vortices become more and more shifted towards the free stream. In the large wavenumber limit, it is found that for small crossflows, Görtler vortices are confined to a thin later centred at a second order turning point distinct from the singular point. As the crossflow is increased towards a critical value, the second order turning point moves towards the singular point. As teh crossflow is increased across the critical value, the second order turning point first coincides with the singular point and then splits into two distinct first order turning points, one of them remaining coincident with the singular point. For crossflow above this critical value, G"ortler vortices are always trapped in a thin region adjacent to the singular point. For large crossflow values, the growth rate is inversely proprotional to the crossflow. Thus a crossflow of comparable size with the streamwise flow has a strong stabilizing effect on inviscid Görtler voritces. The results we find here for hypersonic fows are in sharp contrast with those results found by Bassom and Hall (1991) for incompressible flows where even a crossflow as small as of order $R^{-1/2}$ times the streamwise flow could destroy Görtler vortices completely.

3 RESULTS FOR SUBSONIC FLOWS

Note here that most of the work discussed in this section though carried out for subsonic flows is equally relevant to the hypersonic regime. Note also that the AFOSR contribution to the research discussed in this section relates principally to the use of the computational-printing resources purchased with AFOSR support. Further support of AFOSR relates to travel expenses associated with the presentation of the results at international meetings.

a The destruction of the Görtler vortex mechanism by a crossflow velocity field.

This work is the subject of Basson and Hall (1991) and was the result of joint work between Professor Hall and Dr. Bassom at Exeter Univesity In most boundary layers of practical relevance it is almost always the case that the basic velocity field is three-dimensional. Thus for example the sweep of a wing causes the boundary layer on a swept wing to be three-dimensional. For a subsonic flow it is known that a particularly violent type of instability is the so-called crossflow vortex instability described some years ago for rotating disc flows by Gregory, Stuart and Walke, Phil. Trans(A),1955. In Bassom and Hall show the relationship of this instability to the Görtler vortex instability mechanism. It is found that for sufficiently small crossflow velocities stationary vortex instabilities are exclusively driven by wall curvature. However at a critical crossflow size the crossflow vortex mechanism of Stuart et al appears as the eigenvalue spectrum for inviscid Görtler vortices deforms. As the crossflow is increased even more only inviscid stationary crossflow vortices are possible and the Görtler mechanism is completely destroyed. This is an important result because it means that in many practical situations Görtler vortices are not possible in the presence of a crossflow velocity field. The critical size of the crossflow field required to anhilate the Görtler mechanism is a function of the Mach number. This dependence is currently being mapped out by Professor Hall and Andrew Dando at NASA Langley. We stress the importance of this area because it could well turn out to be the case that the crossflow associated with the external boundary layers associated with the NASP project could cause the Görtler mechanism to be not operational. We note however that in the flows associated with engine inlets the basic state will be almost axisymmetric and so the mechanism will occur.

b The secondary instability of large amplitude vortices to Rayleigh instability waves.

This is joint work between Professor Hall and a research student Nicola Horseman, Hall and Horseman (1991). Here the nonlinear vortex state in a subsonic boundary layer is calculated numerically from the Navier Stokes equations. The instability of this flow to a Rayleigh wave is then considered using a combination of asymptotic and numerical methods. We show that the vortex driven flow is inviscidly unstable to Rayleigh waves and obtain remarkably good agreement between our work and the experimental observations of Blackwelder at USC. This work is important because Rayleigh waves have relatively large growth rates in situations where they occur. In the subsonic case Rayleigh waves can only occur once a longitudinal vortex state has been set up either by curvature effects or as a result of interacting oblique Tollmien-Schlichting waves.

The receptivity problem for Görtler vortices

This work is discussed in Denier, Hall and Seddougui (1991). The central question associated with the receptivity of a boundary layer to Görtler vortices is the question of whether free-stream disturbances or wall roughness can stimulate unstable vortices.

From the work discussed in report 6 and related work by Professor Hall on subsonic flows we know that free-stream disturbances and wall roughness are both highly efficient stimulators of vortices. Professor Hall and Dr. Fu have obtained preliminary results for the receptivity problem for hypersonic Görtler vortices and a paper will be prepared in the near future. Our results suggest that free-stream disturbances are probably the most efficient mechanism for the production of vortices in hypersonic boundary layers. This significant difference from the subsonic case is due to the fact that the most unstable Görtler vortices are located in the transition layer at the edge of the boundary layer. Thus wall-roughness produces a relatively weak response because the vortices decay exponentially in amplitude until the edge of the boundary layer is reached; by contrast free-stream disturbances provoke a bigger vortex response because they are already in the position where instability can occur.

CONFERENCES ATTENDED

Third Symposium on Laminar-Turbulence Transition, Toulouse, France, 11th to 15th September, 1989.
Presentation: The Götler vortex mechanism in compressible boundary layers.

32nd British Theoretical Mechanics Colloquium, University of St., Andrews, April 1990.
Presentation: A linear theory of the Görtler vortex mechanism in hypersonic on Görtler Vortex Flows.

Colloquium on Görtler Vortex Flows, EUROMECH 261, NATO Advanced Research Workshop, Nantes, France, June 11-14, 1990.
Presentation: A linear theory of the Görtler instability in hypersonic boundary layers.

IUTAM Symposium on Nonlinear Hydrodynamic Stability and Transition CNRS-University of Nice and Sophia-Antipolis, France, Sept. 3-7, 1990.
Presentation: Nonlinear development and secondary instability of large amplitude Görtler vortices in hypersonic boundary layers.
33rd British Applied Maths Colloquium, Oxford University, 9-12 April 1991.
Presentation: Görtler instability mechanism in hypersonic boundary layers: linear theory, nonlinear theory and secondary instability.

Applied Maths Seminar, University of East Anglia, Norwich, May 1991.
Presentation: Görtler instability mecganism in hypersonic boundary layers.

Applied Maths Seminar, University of Manchester, February, 1991.
Presentation: Görtler instability mechanism in hypersonic boundary layers.

Applied Maths Seminar, University College of London, February, 1992.
Presentation: Crossflow effects on the growth rate of inviscid Görtler vortices in a hypersonic layer.

PUBLICATIONS

1. P. Hall and Y.B. Fu, On the Görtler vortex instability mechanism at hypersonic speeds, *Theoret. Comput. Fluid Dynamics*, **1** (1989), 125-134.
2. P. Hall and Y.B. Fu, On the Görtler vortex mechanism in ciompressible boundary layers, in *Proceedings of the Third Symposium on Laminar-Turbulent Transition*, 453-463, Toulouse, France, 1989.
3. Y.B. Fu and P. Hall, Nonlinear development and secondart instability of large amplitude Görtler vortices in hypersonic boundary layers, in *Proceedings of the IUTAM Symposium on Nonlinear Hydrodynamic Stability and Transition*, 283-288, Nice, France, 1990.
4. Y.B. Fu, P. Hall and N. Blackaby, On the Görtler instability in hypersonic flows: Sutherland law fluids and real gas effects, NASA ICASE report 90-85 and to appear in *Phil. Trans. Royal Soc. Lond.*.
5. Y.B. Fu and P. Hall, Nonlinear development and secondary instability of large amplitude Görtler vortices in hypersonic boundary layers, an expanded version of [3]., NASA ICASE report 91-39 and to appear in *Eur. J. Mech., B/Fluids*, 1992.
6. Y.B. Fu and P. Hall, Effects of Görtler Vortices, Wall Cooling and Gas Dissociation on the Rayleigh Instability in a Hypersonic Boundary Layer, submitted to *J. Fluid Mech.*, 1992.
7. Y.B. Fu and P. Hallm Crossflow effects on the growth rate of inviscid Görtler vortices in a hypersonic boundary layer, submitted for publication.

8. Y.B. Fu, P. Hall and N.D. Blackaby, On the Görtler Instability in Hypersonic Flows: Sutherland Law Fluids and Real Gas Effects. ICASE report 90-85, December 11, 1990, 70 pages. Accepted for publication in *Proc. Roy. Soc.*

9. J.P. Denier and P. Hall, The Effect of Wall Compliance on the Görtler Vortex Instability. ICASE report 90-84, December 6, 1990, 15 pages. To appear in *Physics of Fluids.*

10. A.P. Bassom, and P. Hall, Vortex Instabilities in 3D Boundary Layers: The Relationship between Görtler and Crossflow Vortices. ICASE report 90-72, October 11, 1990, 76 pages. *Journal of Fluid Mechanics*, 1991, **232**, 312-340.

11. P. Hall and N.J. Horseman, The Inviscid Secondary Instability of Fully Nonlinear Longitudinal Vortex Structures in Growing Layers. ICASE report 90-71, October 10, 1990, 41 pages. *Journal of Fluid Mechanics*, 1991, **3323**, 357-375.

12. N. Blackaby, S. Cowley and P. Hall, On the Instability of Hypersonic Flow Past a Flow Plate. ICASE report 90-40, May 30, 1990, 52 pages. To appear in *Journal of Fluid Mechanics.*

13. J.P. Denier, P. Hall abd S. Seddougui, On the Receptivity Problem for Görtler Vortices: Vortices Motions Induced by Wall Roughness. ICASE report 90-31, May 8, 1990, 59 pages. *Phil. Trans. R. Soc. Lond. A.*, 1991, **335**, 57-85.